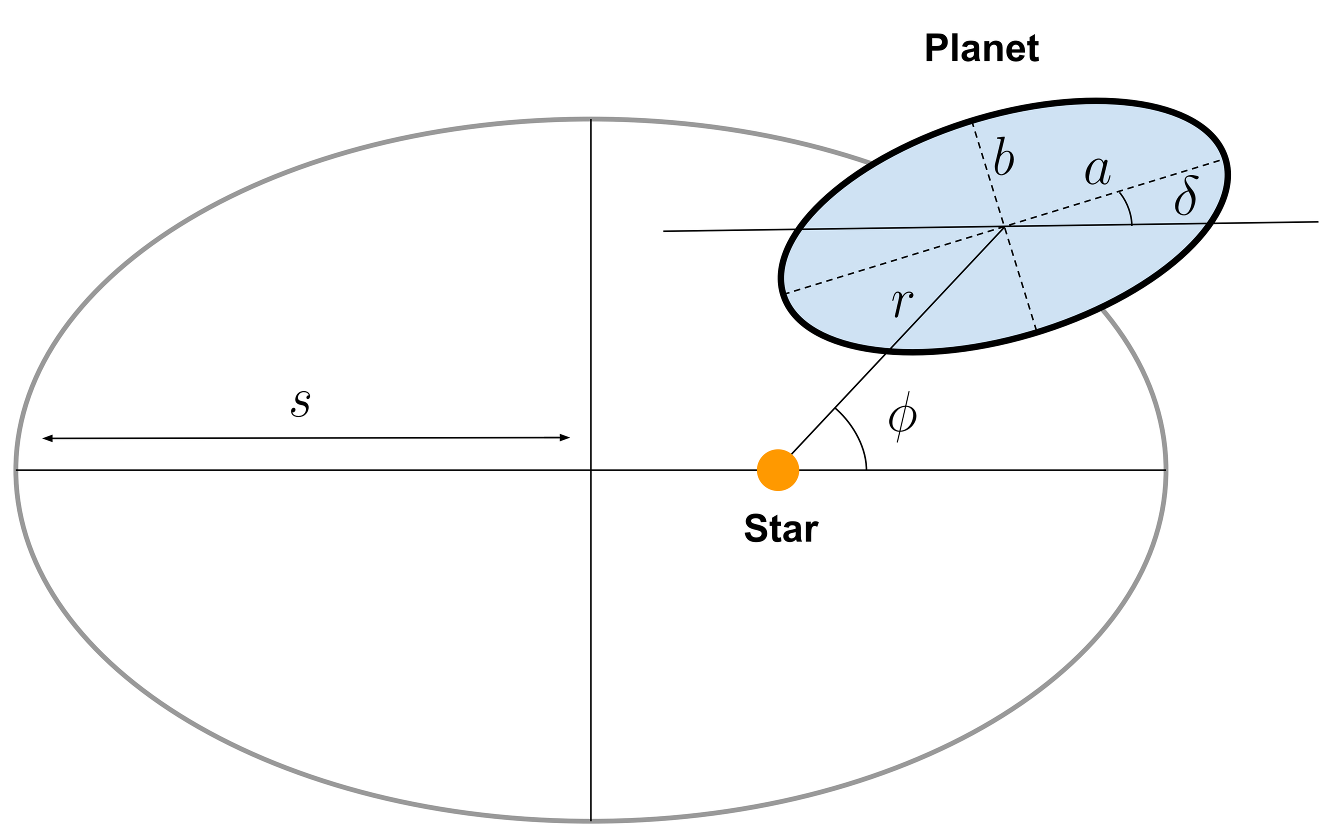
Spin-Orbit Gravitational Locking

Christopher Clouse | Mentor: Miguel Fiolhais | CUNY Research Scholars Program | Borough of Manhattan Community College, City University of New York

1 Introduction

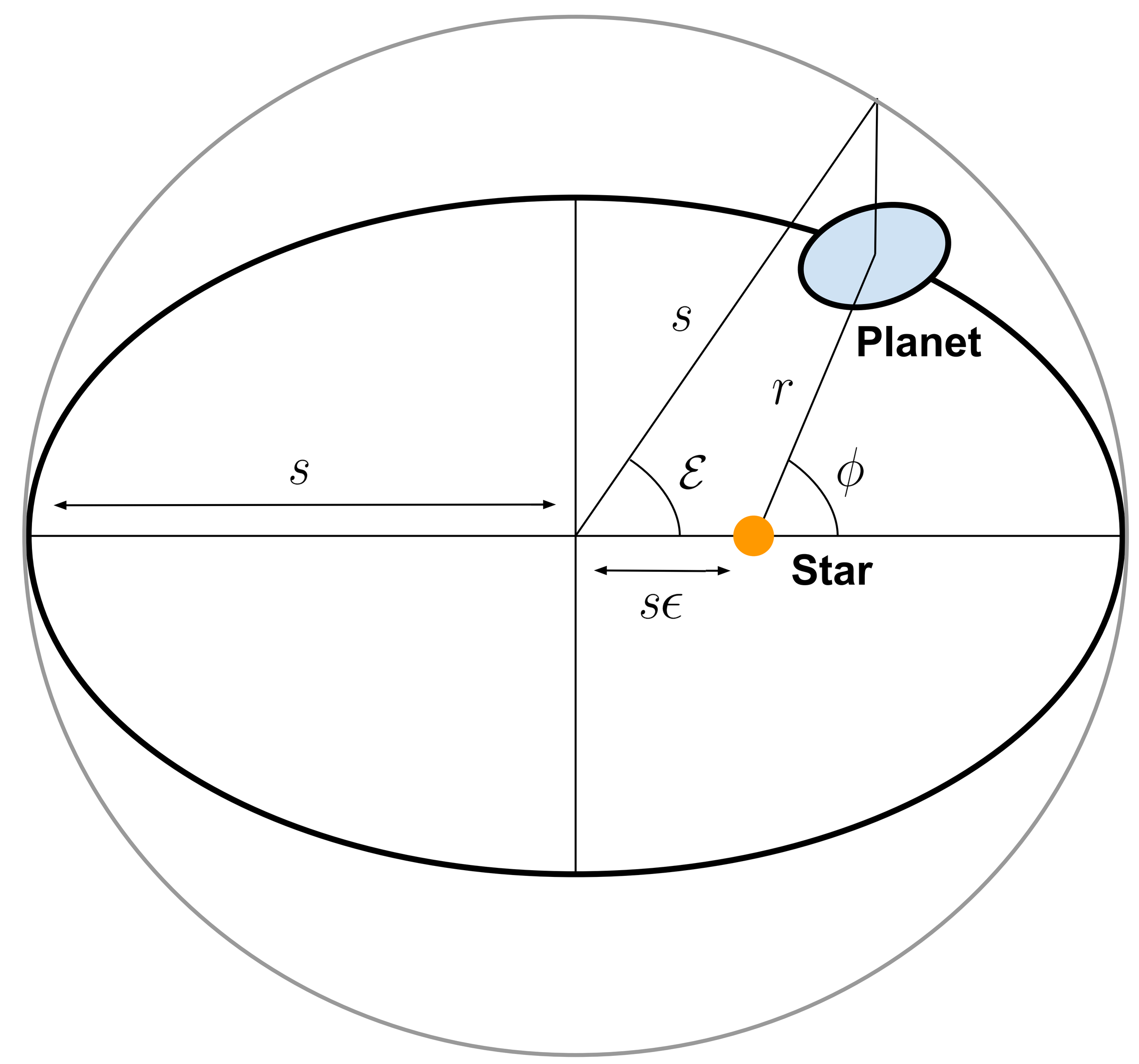
In a tidally locked two-body system, the orbital angular velocity of the objects around the common center of mass of the system is equal to the angular speed of one or both objects spinning around their own axes. The most noticeable example is the case of the Moon orbiting around planet Earth. In this case, the Moon rotates in an approximately circular orbit around the center-of-mass of the Earth-Moon system in exactly the same time as it takes to revolve around its axis. Consequently, the near side of the Moon is always facing the Earth, while the far side is always hidden from an Earthling's view. The tidal friction resulting from the bulges produced by the gravitational force of the Moon on Earth's crust will eventually dissipate energy and slow down Earth's rotation until the system is completely tidally locked.



Despite the fact that tidal locking had already been studied in some detail, in 1964 the Arecibo Telescope revealed a new manifestation of a closely related phenomenon. The rotation period of Mercury around its own axis was only 59 days, as opposed to its 88-day orbital period - an approximate 3:2 spin-orbit resonance. The reason for this anomalous behavior was soon identified as stemming from the ellipsoidal shape of Mercury and its high eccentricity orbit around the Sun. The objective of this work is to study the rotational potential energy of an ellipsoidal planet orbiting a spherical star on an elliptical orbit with fixed eccentricity and semi-major axis, in order to show that the system presents an infinite number of metastable equilibrium configurations. These states correspond to local minima of the rotational potential energy averaged over an orbit, where the ratio between the period of the planet around its axis and the period around the star are equal to simple rational ratios.

2 Spin-Orbit Dynamics

The figure below shows a planet orbiting a star. The star is modeled as a spherical object, while the planet is an ellipsoid.



The total mechanical energy of a system up to the quadrupole contribution in the gravitational potential can be expressed as

$$E = \frac{1}{2}m\dot{r}^2 - G\frac{M_s m}{r} - \frac{3GM_s}{2r^3}(B-A)\left[\cos^2(\phi-\delta) - \frac{1}{3}\right] + \frac{l^2}{2mr^2} + \frac{1}{2}B\dot{\delta}^2$$

The part of the energy of the system associated to the orbital motion is fixed and it depends on the eccentricity and semi-major axis of the orbit. Therefore, the rotational energy is

$$E_{\text{rot}} = \frac{1}{2}B\dot{\delta}^2 - \frac{3GM_s}{2s^3}(B-A)\left(\frac{1+\epsilon\cos\phi}{1-\epsilon^2}\right)^3\left[\cos^2(\phi-\delta) - \frac{1}{3}\right]$$

By introducing the following angle,

$$\gamma \equiv \delta - pM$$

the Lagrangian of the system can be expressed as,

$$L = \frac{1}{2}B(\dot{\gamma} + pn)^2 - Q\left[S(\epsilon) + R(p,\epsilon)\cos(2(p\pi+\gamma))\sin(2p\pi)\right]$$

in terms of eccentricity, angle, and a parameter p, with,

$$Q \equiv \frac{3GM_s}{2s^3}(B-A)$$

which ultimately leads to the following of equation of motion,

$$B\ddot{\gamma} + QH(p,\epsilon)\sin(2\gamma) = 0$$

for $p = k/2$, where k is an integer number. If the function H is positive, this equation of motion is the same type of the equation of motion for a simple pendulum.

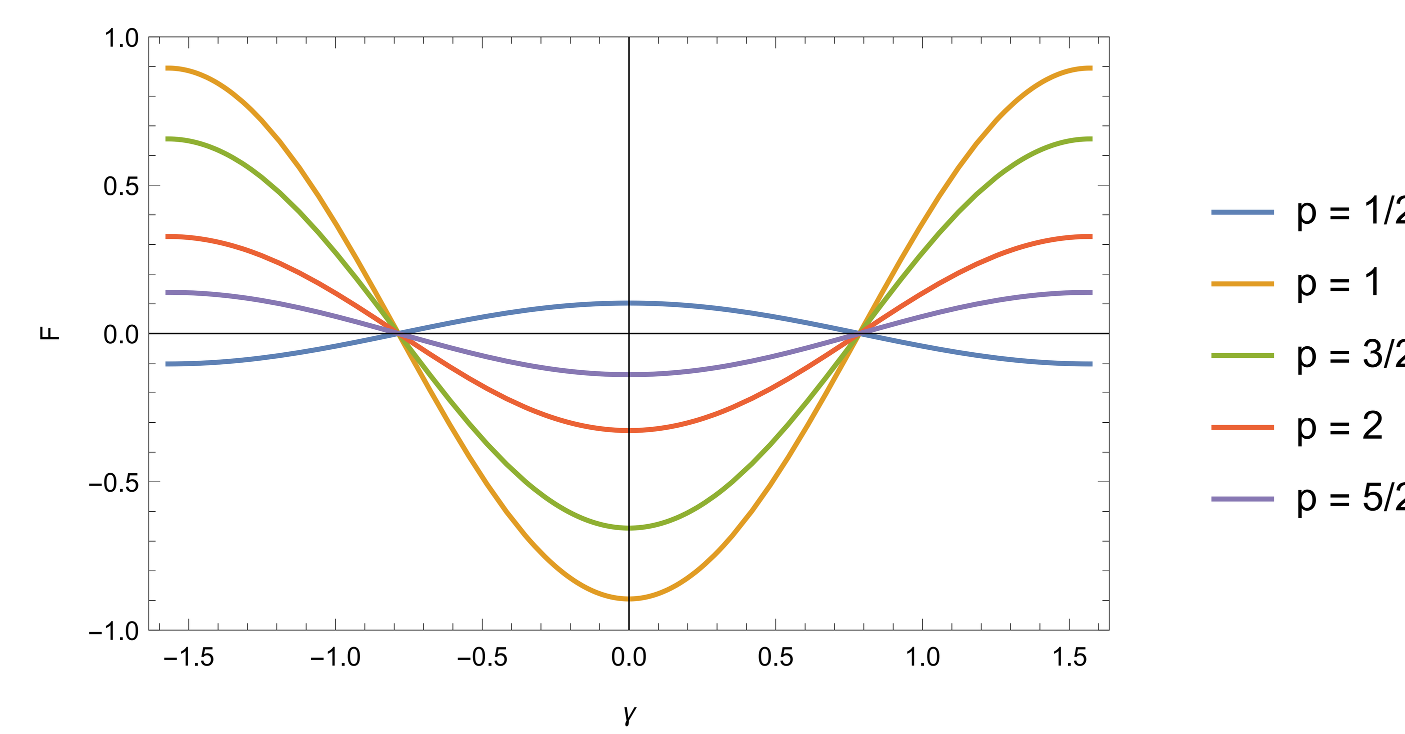
3 Results

According to the equation of motion derived in the previous section, the frequency of small oscillations is,

$$\omega = \sqrt{\frac{2QH(p,\epsilon)}{B}}$$

and the periods of the spin and orbital motions are locked in an integer or half-integer ratio. Therefore, if the longest semi-axis of the planet points toward the star at perihelion, it is again pointing toward the star after two orbital periods.

This phenomenon can also be observed by plotting dependency of the potential energy term with respect to the angular coordinate. The figure below clearly shows that there is a minimum of the potential located at angle zero with the exception of p = 1/2. As expected, the deepest minimum corresponds to p = 1 and the second deepest minimum corresponds to the Mercury spin-orbit resonance, p = 3/2.



4 Conclusion

- It is shown that, for an ellipsoidal planet, there are an infinite number of metastable configurations in which the planet rotates around its axis an integer number of times for every two revolutions around the star.
- The second energetically most favored spin-orbit resonance corresponds to the planet spinning three times around its axis for every two orbits around the star, such as the orbit of Mercury around the Sun.

5 References

- P. Goldreich & S. Peale, AJ **71**, 425 (1966).
- A. Ferroglia & M.C.N. Fiolhais, AJP **88**, 1059 (2020).