DESIGNING BRIDGES WITH UNUSUAL GEOMERTIES

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Introduction

The purpose of this project is to be able to predict the shape of a suspension bridge's suspension cable for bridges which have unusual geometries. Unusual geometries meaning bridges lacking symmetries, for example in the deck's weight distribution and/or tower heights.

We used differential equations to model and predict the suspension cable height. The equations are gauged towards taking variables into account that can be determined during the design phase..

Clifton Suspension Bridge – Bristol ,England

Project

We will be drafting two different bridges. We will call the first bridge, "Bridge 1". Bridge 1 has a classical symmetrical design to test the methods and code we will be use using.

We will also have "Bridge 2". Bridge 2 has an unusual non-symmetric geometry.

We will look into forces such as tension, or a pull force, that the cable, road and towers experience. Weight, density, thickness, and gravity, all play into tension forces when designing bridges.

The values of the parameters will produce different cable shapes

The suspension cable differential equation that we use is::

$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0}$$

$y(x)$ is the height (altitude) of the cable
x is the horizontal distance (from some starting point)
$w(x)$ is the verticle force per unit length on the cable (N/m)
T_0 is the hoizontal tension in the cable.

Method

Using Maplesoft, a mathematical programming language, we drafted code with variables for length, density of concrete, width of the slab, gravity, tower height and so on, which would solve the suspension cable equation for us. 3D models were then rendered in FreeCad (Computer Aided Design Software).

The classical, symmetric case (Bridge 1) is relatively easy to solve:

- **Question.** The bridge deck is flat and of length s. It's weight per unit length is constant w_0 (negligible cable weight). The suspension cable is attached to the towers at a height of h. Find the equation of the suspension cable.

Solution. Integrate $\frac{d^2y}{dx^2} = \frac{w_0}{T_0}$ twice.

$y(x) = \frac{w_0 x^2}{2T_0} + c_1 x + c_0$

The boundary conditions $y(0) = y(s) = h$ imply: $c_0 = h$ and $c_1 = -\frac{w_0 s}{2T_0}$.

By symmetry $y'\left(\frac{s}{2}\right) = 0$, which implies $T_0 = \frac{w_0 s^2}{8h}$.

In textbook treatments of solving the cable equation, symmetry or some knowledge of where the low point of the cable is used to solve for the horizontal tension *To.*

However, solving the same problem for bridges with unusual (non-symmetric geometries is much more involved and requires the use of numerical techniques and optimization routines.

The following method we derived solves the general (non-symmetric) case.

$w(x) = udg(f_1(x) - f_0(x))$
$u =$ density (kg/m^2)
$d =$ thickness (m)
$g =$ acceleration of gravity 9.8 (m/s^2)

- $\frac{d^2y}{dx^2} = \frac{w(x)}{T_0}$

(1) integrate $w(x)$ twice
(2) $y(x) = \int\int \frac{w(x)}{T_0}\,dx\,dx + c_1 x + c_0$
(3) Using tower attachment heights solve for c_1 and c_0 in terms of T_0
(4) For each tension T_0, (numerically) find minimum of $y(x), x \in [0, s]$. Call this function $myT(T_0)$. Note. $myT(T_0)$ is monotonically increasing.
(5) Find T_0 so cable low point $myT(T_0)$ is at desired height. (Newton's Method)
(6) Find x coord of low point of cable. Newton or any lazy algorithm as $y(x)$ is concave up.
(7) Calculate where suspender cables are attached.

"Deck span"
$S := 10$
"Height of left tower"
$y0 := 5$
"Height of right tower"
$yS := 10$
"altitude of top of bridge deck"
$f1 := x \mapsto 0$
"altitude of bottom of bridge deck"
$f0 := x \mapsto -\frac{(x-10)^2}{25} - 1$
"T0 horizontal tension in cable needed"
47198.67189
"x location of lowest point of cable"
$botx := 3.450000000$
"Suspension Cable Equation giving altitude of suspension cable as function of x"
$0.1245797766x^2 + 0.0008305318441(x - 10.)^4 + 0.08473407800x - 3.305318440$
$NumAttachmentsR := 6$
"x coordinates of suspender cable attachment points to the right of lowest point of cable"
$deckAttachmentPointsR :=$ 4.450000000, 5.450000000, 6.450000000, 7.450000000, 8.450000000, 9.450000000
"y coordinates of suspender cable attachment points to the right of lowest point of cable"
$suspenderAttachmentPointsR :=$ 0.326742868, 1.212773654, 2.555954083, 4.275556460, 6.310785853, 8.620780100
$NumAttachmentsL := 3$
"x coordinates of suspender cable attachment points to the left of lowest point of cable"
$deckAttachmentPointsL :=$ 2.450000000, 1.450000000, 0.450000000
"y coordinates of suspender cable attachment points to the left of lowest point of cable"
$suspenderAttachmentPointsL :=$ 0.348704828, 1.517816785, 3.666316808

Dimensions for Bridge 2 calculated by Maple.

Results

Our method, implemented in the Maple Software, was utilized to produce procedural generated graphs. This means, with influence from the variables input, the solution and result will vary accordingly, being a sign of modularity in visualizing the geometry and dimensions. We had used two sets of parameters, producing two different graphs. The rendered graphs have a measurable horizontal distance of the deck, height of the towers, and an arc we can interpret as the shape of their cables. We can further interpret their decks width and suspender locations. Bridge 1 has a symmetrical output, where applied loads will be distributed evenly across the structure between all frames and joints of the bridge. The second graph has the towers at different heights, a deck shape that extends further downward on the shorter tower and slopes to be thinner at the taller tower. This resulting image changes depending on the variables input, showing that it can be modular in giving dimensions from the original equation we have stemmed from.

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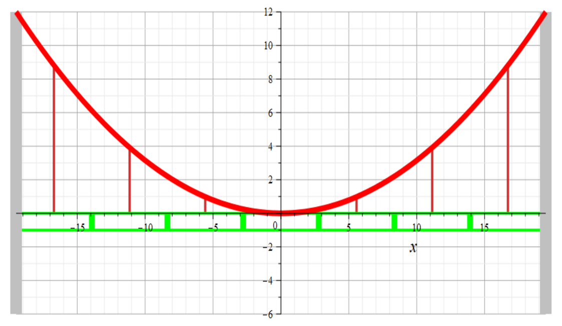
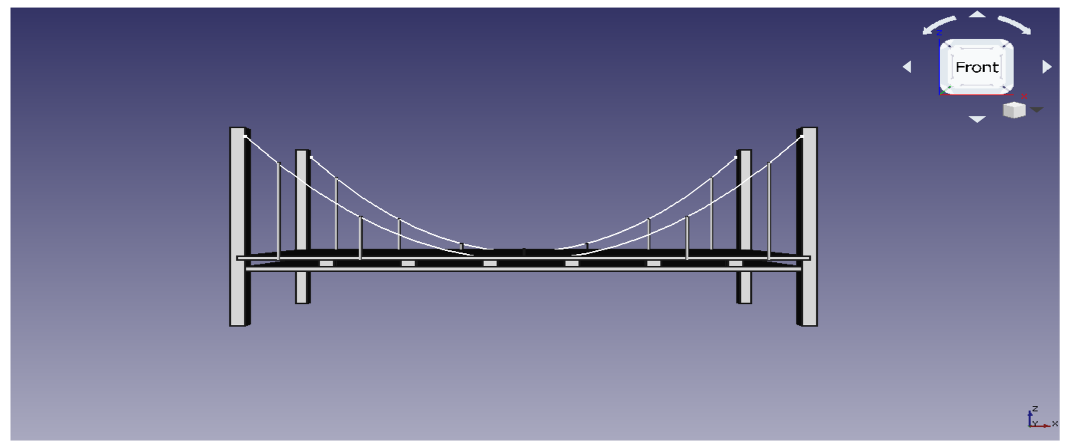
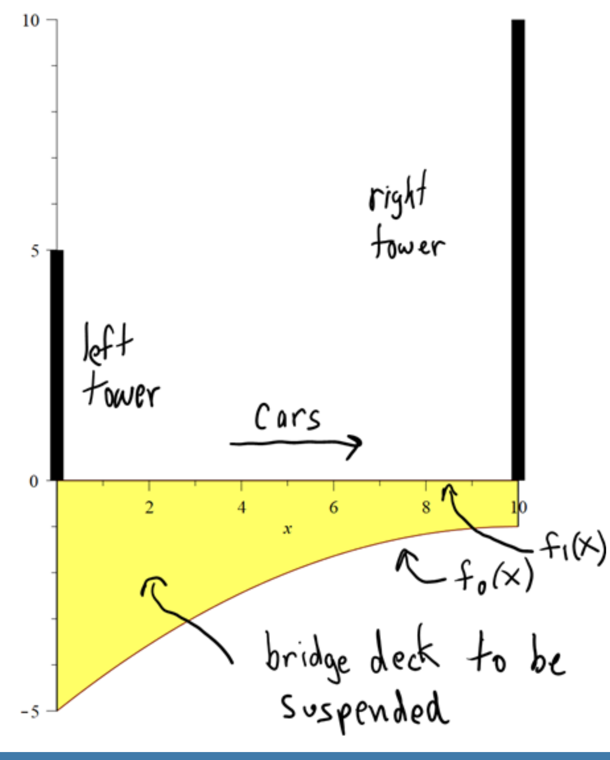


Bridge 1 rendered in Maple

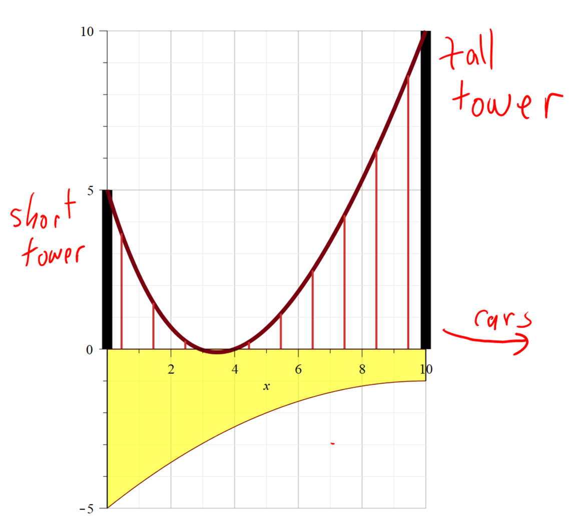


Bridge 1 rendered in FreeCad

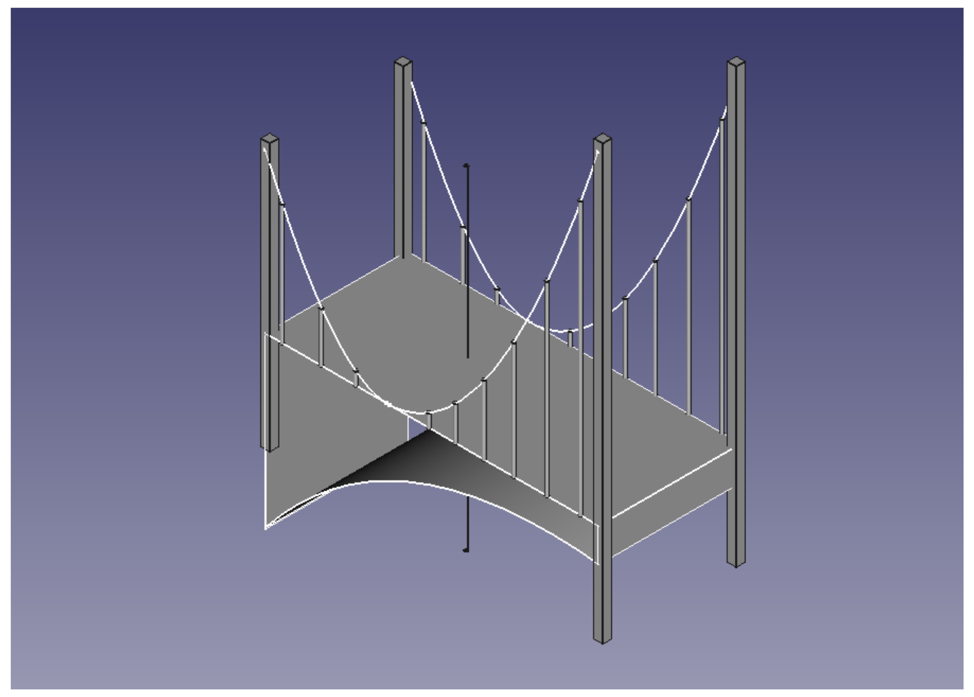
Bridge 2 Design Parameters



Bridge 2 Solution (rendered in Maple)



Bridge 2 Rendered in FreeCad



Conclusion

In using the derived cable equation, we can take away that the bridge is centered around the cable shape. It is complicated to use despite that is is relatively simple in obtaining a solution to design a bridge off. From the CAD render, we have visualized a model (first represented in Maple) that aids in verifying the shape of the bridges do have reasonable dimensions. To further show the proof off the generated images, building a physical model would aid in showcasing the shape of the models in a physical space in verifying the dimensions.

$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0}$$

Bibliography

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