4.3 Logarithmic Functions

and

4.4 Graphs of Logarithmic Functio

$f(x) = b^x$ is a one-to-one function.

$f(x)$ has an inverse functio

it is called:

"The Logarithmic Function with base b."

if $2^3 = 8$

then $\log_2 8 = 3$

Definition

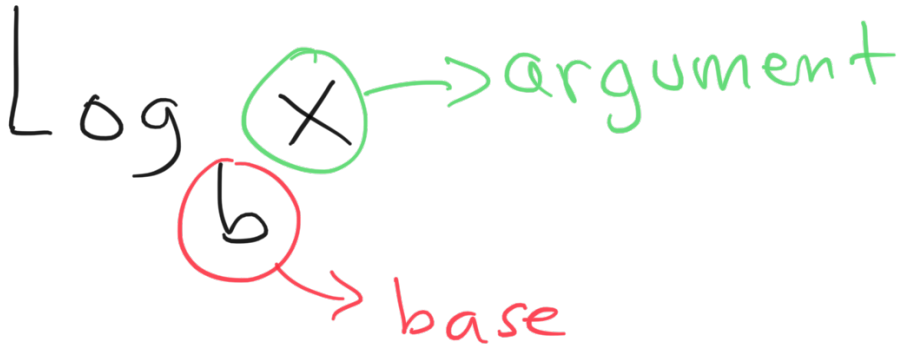
For $x > 0$, $b > 0$, and $b \neq 1$

$y = \log_b x$ if $x = b^y$

The function

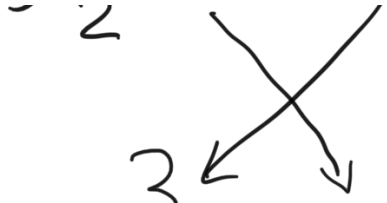
$f(x) = \log_b x$ (Read "log base b of x")

is the logarithmic function with base b



Example:

$\log_2 8 = 3$ (logarithmic



The logarithmic function with base 10 is called the common logarithmic function. It is denoted by $\log_{10}$ or simply Log.

Evaluating Common Log

Examples: Evaluate

Given $f(x) = \text{Log } x$ evalu

(round to 3 dec. places)

$10^2 = 1$

a) $f(100) = 2$ $\quad \log_{10}(100)$

b) $f\left(\frac{1}{3}\right) = -0.477$

c) $f(-2) =$ Undefined

→ the inside of th
Log always positiv

Fact:

Logarithmic functions are "one to one"

Examples: Solve

$$\log(2x+1) = \log(3x)$$

Solution:

$$\log_{10}(2x+1) = \log_{10}(3x)$$

$$2x+1 = 3x$$

$-2x$ $-2x$

$1 = x$

Now check

$x = 1$

$\log(2(1)+1) \stackrel{?}{=} \log(3 \cdot 1)$

$\log(3) \stackrel{\checkmark}{=} \log 3$

Then solution $x = 1$

Example 2

Solve $\log_4(x^2-6) = \log_4 10$

Solution:

$\log_4(x^2-6) = \log_4 10$

$x^2 - 6 = 10$

$+6 \quad +6$

model

eve

$x^3 = 8$

$x = \sqrt[3]{8}$

$x = 2$

$\sqrt{x^2} = \sqrt{16}$

$x = \pm\sqrt{16}$

$x = \pm 4$

$x = 4$ $\qquad$ $x = -4$

$x^2 = 16$

$4^2 = (4)(4)$

16

$(-4)^2 = (-4)(-4)$

16

Check $x = 4$ ✓

$\log_4((4)^2 - 6) \stackrel{?}{=} \log_4 10$

$\log_4(10) \stackrel{?}{=} \log_4 10$ ✓

Check $x = -4$ ✓

$\stackrel{?}{=}$

$\log_4((-4)^2 - 6) = \log_4 10$

$\log_4(16 - 6) \overset{?}{=} \log_4 10$

$\log_4(10) \overset{\checkmark}{=} \log_4 10$

The Solutions are

$X = 4$ and $X = -4$

Solution Set $= \{4, -4\}$