4.1 Exponential Functions

and

4.2 Graphs of Exponential Function

So far we have seen functions like

$f(x) = x^2$ → base

$g(x) = x^3$

However it possible to form functions like

$f(x) = 2^x$ → base

$g(x) = 3^x$ →

$g(x) = (3)^x$

$f(x)$ and $g(x)$ are called exponential functions

Definition

The exponential function f with base b is denoted by

$$f(x) = b^x$$

where $b > 0$, $b \neq 1$

and x is any real number

Examples:

Example

$f(x) = 5^x$

$g(x) = \left(\frac{1}{2}\right)^x$

$h(x) = (0.7)^x$

are exponential functions

Evaluation of Exponential Functions

Example 1

Evaluate $f(x) = 2^x$ for $x = -3.1$ (Round to 3 decimal places)

Solution:

$$f(-3.1) = 2^{-3.1}$$

2 [^] [(-)] 3.1 [=]

$$= 0.1166291 \approx 117$$

+1

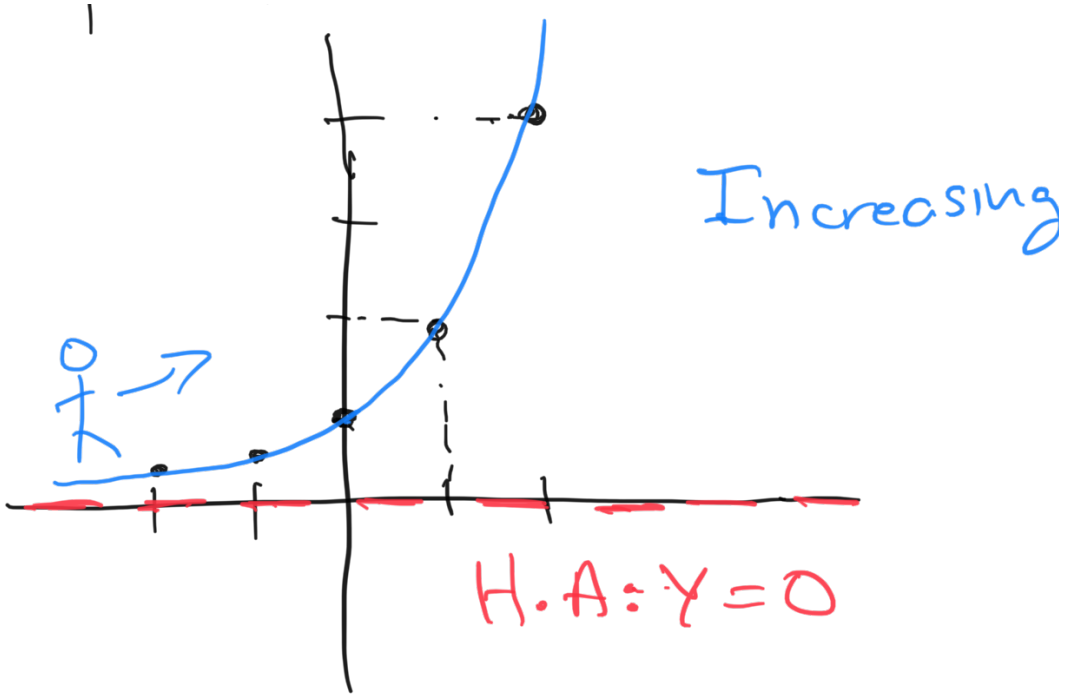
Example 2 go to worksheet

Graph of Exponential Function

Graph of $y = b^x$ where $b > 1$

Example: Graph $f(x) = 2^x$

x	-2	-1	0	1	2
2^x	0.25	0.5	1	2	4

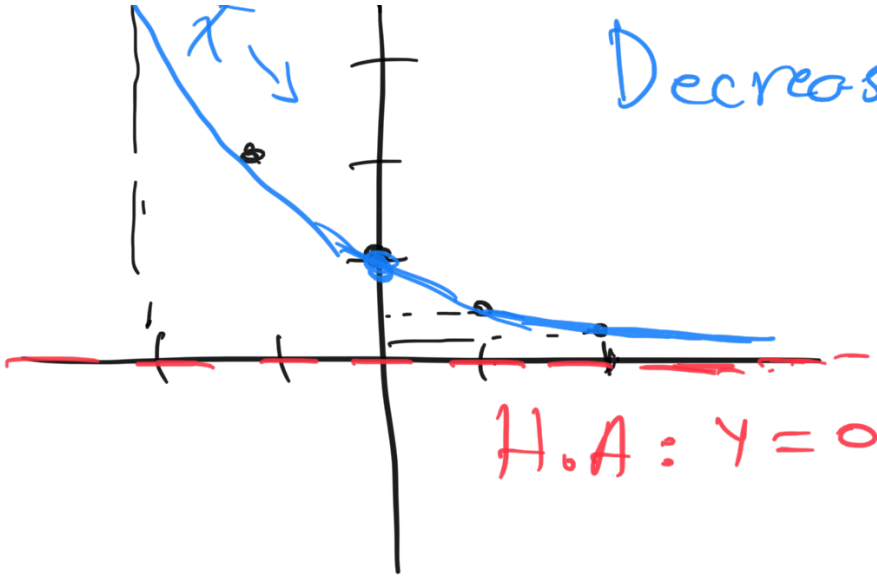


Graph of $Y = b^x$

where $0 < b < 1$

Example: Graph $Y = \left(\frac{1}{2}\right)^x$

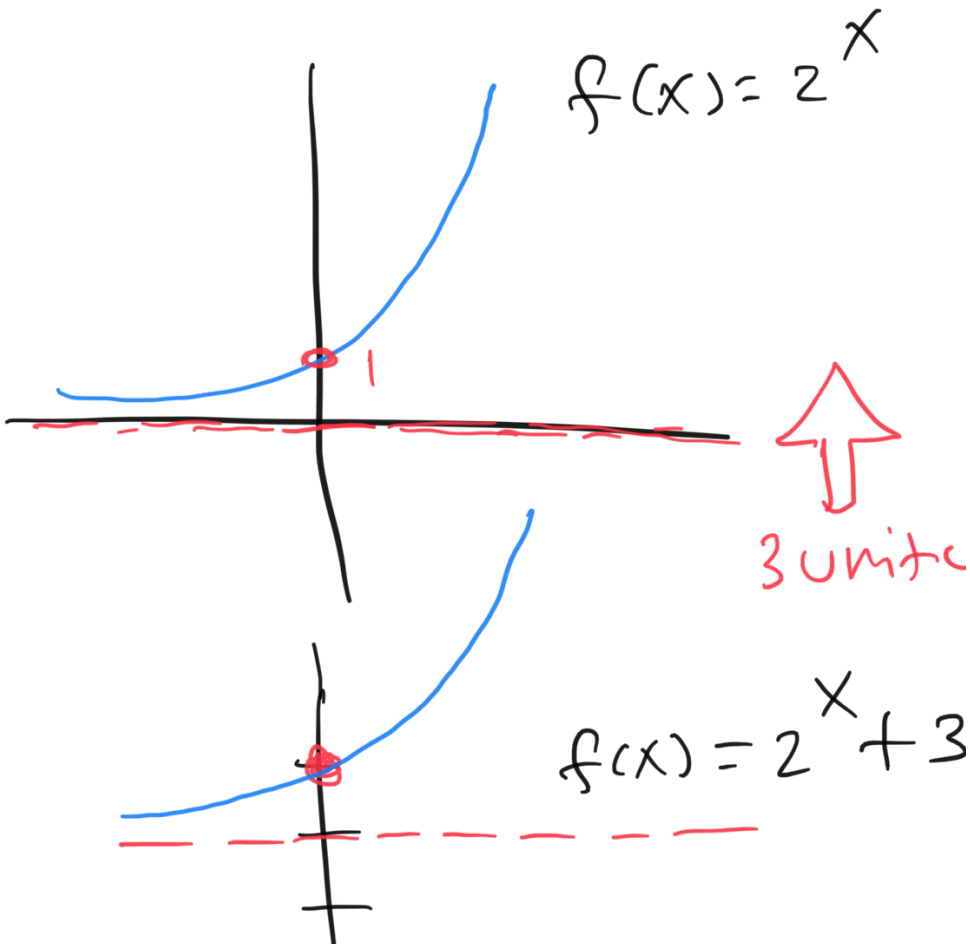
X	-2	-1	0	1	2
$\left(\frac{1}{2}\right)^x$	4	2	1	0.5	0.25

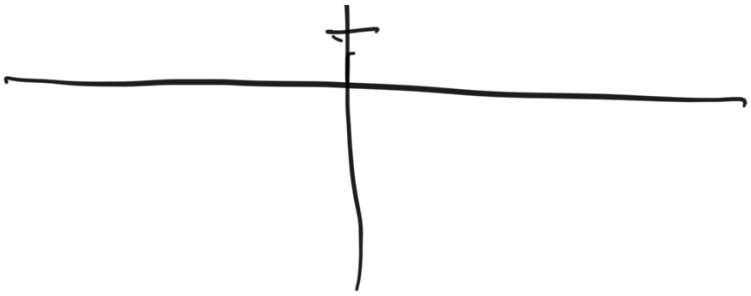


Example: Graph

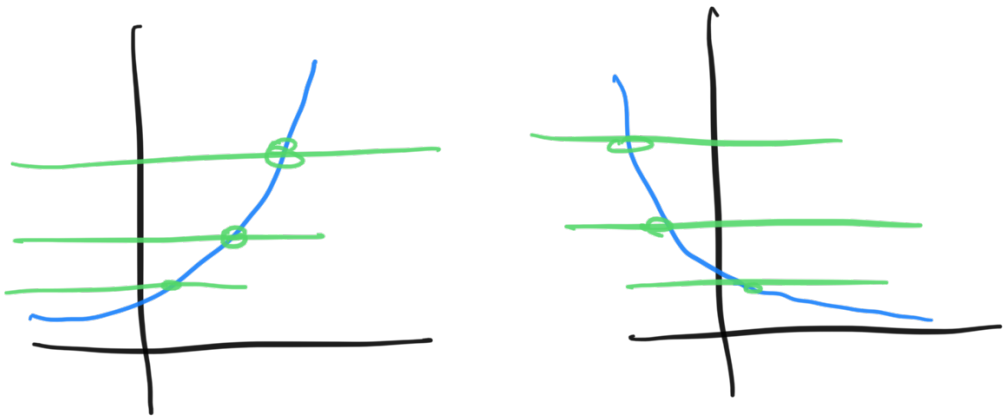
$$f(x) = 2^x + 3$$

Sol:





Exponential Function are one-to-one functions because they pass the horizontal line test.



Using the one to one property to solve

exponential equations

Example 1:

Solve $9 = 3^{2x-4}$

Sol:

write it as a power of

$9 = 3^{2x-4}$

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3 = 3$

$2 = 2x - 4$

$+4 \quad +4$

$\frac{6}{2} = \frac{2x}{2}$

$3 = x$

Example 2: Solve

x

$$\left(\frac{1}{2}\right) = 8$$

Solution:

Let's express left and right side using base 2

$$(2^{-1})^{x} = 8$$

$$2^{-x} = 8$$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$$2^{-x} = 2^{3}$$

$$-x = 3$$

$$\frac{-x}{-1} = \frac{3}{-1}$$

$x \approx -3$

The Natural Base e (Euler's Number)

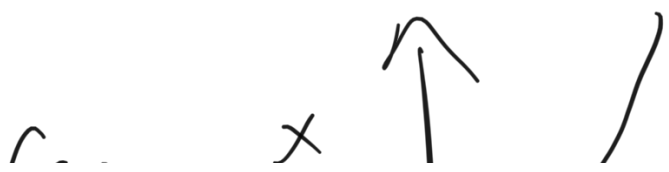
In many applications, the most convenient base is

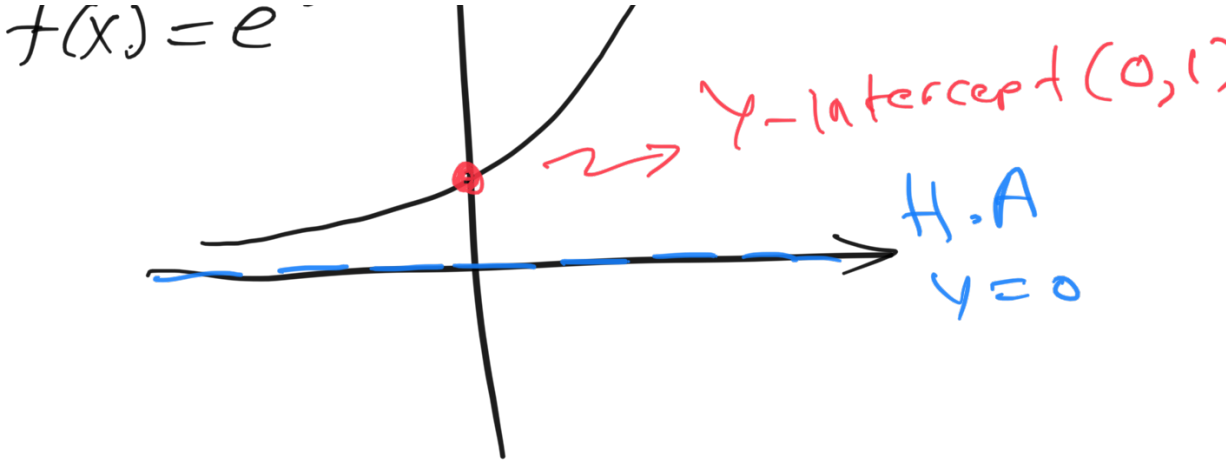
$e \approx 2.718281828\ldots$

e → Natural base

$f(x) = e^x$ is called "The natural exponential Function"

Since $e > 1$





Let $f(x) = e^x$ compute

$f(3) = 20.0855$ e^(3)

$f(-5) = 0.0673$ e^(-5

$f(2.8) = 16.445$ e^(2.8

Applications of Exponential function.

Formulas for Compound Interest

Compound interest is calculated

Compound

by the formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{n \times t}$$ where

$A(t)$ = amount after t years

P = Principal

r = interest per year (in Decimal form)

n = number of times interest is compounded per year

t = # of years

Compounded	n
Annually	1
Semiannually	2
Quarterly	4

monthly	12
Daily	365

Continuously compounded interest is calculated by the formula

$$A(t) = Pe^{rt}$$ where

$A(t)$ = amount after t year

P = principal

r = interest rate per year (in Decimal form)

t = # of years