Sum and Difference Identities

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the exact value of the expression.

1) $\cos(60° + 45°)$ 1) _____

A) $\frac{\sqrt{2} + 2\sqrt{3}}{4}$ B) $\frac{\sqrt{2} - \sqrt{6}}{4}$ C) $\frac{\sqrt{6} - \sqrt{2}}{4}$ D) $\frac{2\sqrt{2} + \sqrt{6}}{4}$

2) $\cos(45° - 30°)$ 2) _____

A) $\frac{\sqrt{6} - \sqrt{2}}{4}$ B) $\frac{\sqrt{2} + \sqrt{6}}{2}$ C) $\frac{\sqrt{6} - \sqrt{2}}{2}$ D) $\frac{\sqrt{2} + \sqrt{6}}{4}$

3) $\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$ 3) _____

A) $\frac{2\sqrt{2} + \sqrt{6}}{4}$ B) $\frac{\sqrt{6} - \sqrt{2}}{4}$ C) $\frac{\sqrt{2} + 2\sqrt{3}}{4}$ D) $\frac{\sqrt{2} - \sqrt{6}}{4}$

4) $\sin 75°$ 4) _____

A) $\frac{2\sqrt{2} + \sqrt{6}}{4}$ B) $\frac{\sqrt{2} + \sqrt{6}}{4}$ C) $\frac{\sqrt{6} - \sqrt{2}}{4}$ D) $\frac{\sqrt{2} + 2\sqrt{3}}{4}$

5) $\sin 15°$ 5) _____

A) $\frac{\sqrt{2}(\sqrt{3} + 1)}{4}$ B) $-\frac{\sqrt{2}(\sqrt{3} - 1)}{4}$

C) $-\frac{\sqrt{2}(\sqrt{3} + 1)}{4}$ D) $\frac{\sqrt{2}(\sqrt{3} - 1)}{4}$

Find the exact value under the given conditions.

6) $\tan\alpha = \frac{3}{4}$, $\pi < \alpha < \frac{3\pi}{2}$; $\cos\beta = -\frac{8}{17}$, $\frac{\pi}{2} < \beta < \pi$ Find $\sin(\alpha + \beta)$. 6) _____

A) $\frac{77}{85}$ B) $-\frac{36}{85}$ C) $\frac{84}{85}$ D) $-\frac{13}{85}$

Use the given information to find the exact value.

7) $\cos A = \frac{1}{3}$, $0 < A < \frac{\pi}{2}$; $\sin B = -\frac{1}{2}$, $\frac{3\pi}{2} < B < 2\pi$ Find $\sin(A - B)$. 7) _____

A) $\frac{\sqrt{3} - 2\sqrt{2}}{6}$ B) $\frac{\sqrt{3} + 2\sqrt{2}}{6}$ C) $\frac{2\sqrt{6} + 1}{6}$ D) $\frac{2\sqrt{6} - 1}{6}$

Use the appropriate sum or difference identity to write the given expression as a function of x alone.

8) $\tan(x - \pi)$ 8) _____

A) $-\tan x$ B) $\frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x}$ C) $\frac{1 + \sqrt{3}\tan x}{\sqrt{3} - \tan x}$ D) $\tan x$

9) $\sin(x - \pi)$ 9) _____

A) $-\cos x$ B) $\sin x$ C) $-\sin x$ D) $\cos x$

10) $\sin\left(\frac{\pi}{2} - x\right)$ 10) _____

A) $\sin x$ B) $\cos x$ C) $-\sin x$ D) $-\cos x$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Establish the identity.

11) $\cos\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$ 11) _____

12) $\sin\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\sin x - \cos x)$ 12) _____

13) $\tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - 1}{1 + \tan x}$ 13) _____

14) $\tan\left(\frac{\pi}{2} + x\right) = -\cot x$ 14) _____

15) $\frac{\sin(\alpha - \beta)}{\sin \alpha \sin \beta} = \cot \beta - \cot \alpha$ 15) _____

16) $\frac{\cos(\alpha + \beta)}{\cos \alpha \sin \beta} = \cot \beta - \tan \alpha$ 16) _____

17) $\sin(x + y) - \sin(x - y) = 2 \cos x \sin y$ 17) _____

18) $\cos(x - y) - \cos(x + y) = 2 \sin x \sin y$ 18) _____

1) B
2) D
3) D
4) B
5) D
6) B
7) C
8) D
9) C
10) B

11) $\cos\left(x + \frac{\pi}{6}\right) = \cos x \cos\frac{\pi}{6} - \sin x \sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x.$

12) $\sin\left(x - \frac{\pi}{4}\right) = \sin x \cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos x = \frac{\sqrt{2}}{2}(\sin x - \cos x).$

13) $\tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - \tan \pi/4}{1 + (\tan x)(\tan \pi/4)} = \frac{\tan x - 1}{1 + \tan x}.$

14) $\tan\left(\frac{\pi}{2} + x\right) = \frac{\sin((\pi/2) + x)}{\cos((\pi/2) + x)} = \frac{\sin(\pi/2)\cos x + \sin x \cos(\pi/2)}{\cos(\pi/2)\cos x - \sin(\pi/2)\sin x} = \frac{1 \cdot \cos x + \sin x \cdot 0}{0 \cdot \cos x - 1 \cdot \sin x} = -\cot x.$

15) $\frac{\sin(\alpha - \beta)}{\sin\alpha \sin\beta} = \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\sin\alpha\sin\beta} = \frac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} - \frac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta} = \frac{\cos\beta}{\sin\beta} - \frac{\cos\alpha}{\sin\alpha} = \cot\beta - \cot\alpha$

16) $\frac{\cos(\alpha + \beta)}{\cos\alpha\sin\beta} = \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\sin\beta} = \frac{\cos\alpha\cos\beta}{\cos\alpha\sin\beta} - \frac{\sin\alpha\sin\beta}{\cos\alpha\sin\beta} = \frac{\cos\beta}{\sin\beta} - \frac{\sin\alpha}{\cos\alpha} = \cot\beta - \tan\alpha$

17) $\sin(x + y) - \sin(x - y) = \sin x \cos y + \cos x \sin y - \sin x \cos y + \cos x \sin y = 2\cos x \sin y.$

18) $\cos(x - y) - \cos(x + y) = \cos x \cos y + \sin x \sin y - (\cos x \cos y - \sin x \sin y) = 2 \sin x \sin y.$