ECONOMETRICS

Conditional volatility model

Introduction

The study involved the prices of shares in the stock exchange market from 2007-01-03 till 2018-4-30. The researcher selected a sample size of 2582 for the study. The prices' opening, closing, high and low values were recorded. Also, the researcher collected volumes of sales of the shares. The data entry and cleaning were done in excel and then exported to R statistical software for analysis. The data was converted into time-series data in R, ready for analysis. The study aimed at obtaining the returns and fitting the generalized autoregressive conditional heteroskedasticity (GARCH) and autoregressive conditional heteroskedasticity (ARCH). Those are the conditional vitality models.

Results and findings

Prices

The share prices were plotted using the charSeries function in R. The data series shows that the prices were high at the start of 2008 (around 35) but dropped significantly to the least price in the series of around 16 at the end of the year. The highest prices were witnessed around the mid of 2016, with a price value of above 40. The plot of the prices of the shares demonstrated an increasing trend, meaning that it was not stationary. The variance was not constant over the series; thus, there was the presence of heteroskedasticity. Figure 1 below shows how the prices of shares were distributed.

Figure 1: Time series plot of prices of shares from 2007 to 2018

Returns

The returns are the changes in the prices of investment, asset, or a project over time. It can be represented by the real values of the price change or percentage. A positive return indicates a profit was made, while a negative return indicates a loss. The returns of the share prices were estimated using dailyReturn in R as shown in the appendix. The returns indicated a maximum profit of 14% and a maximum loss of 13% over the period. The average and standard deviations of the returns were 0. The researcher plotted returns for easy interpretation and observed that normality existed. Figure 2 below shows that the mean of the returns was constant all through at zero. Mid 2013 was associated with the maximum loss while mid-2014 was associated with the maximum profit, as evidenced in the graph below;

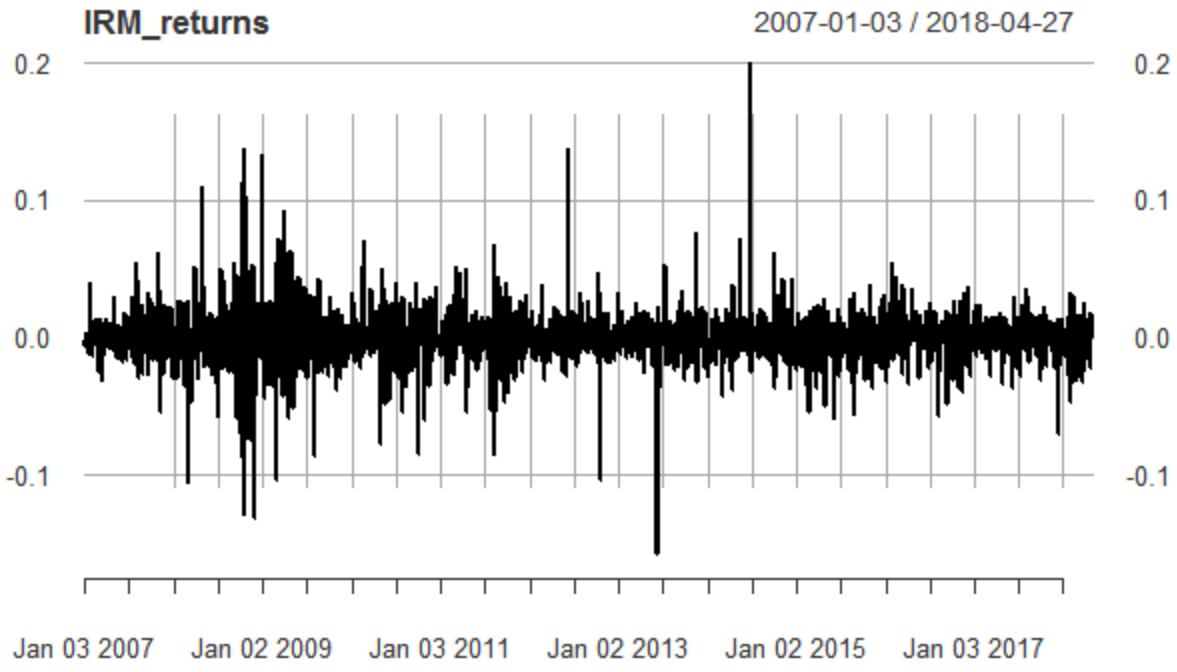

Figure 2: Plot of returns of the prices of shares

Model

It is the most used measure of the risk associated with assets and investments. Volatility models are very important when dealing with economic and financial models because of the high fluctuations. In my case, the data was entered daily, showing how fluctuating it was. As a result, GARCH was fitted to aid in understanding the fluctuations and making predictions on the possibility of prices increasing or decreasing. The GARCH model of conditional variance was fitted as GARCH (1, 1) and ARFIMA (1, 0, 1), the mean model, for the returns. The distribution of the returns was normal; thus, the shape, skew, and lambda was not produced. As per the Pearson adjusted goodness of fit statistics, the p-values are less than the alpha at 0.05 test of significance; thus, we conclude that the model was statistically significant. Additionally, the likelihood estimate was obtained to be 7425. 046. The high value of the likelihood estimate

indicates the model's goodness of fit. The optimal parameters of the model were as shown by table 1 below;

Table 1: Optimal parameters of the GARCH model

	Estimate	std. Error	t value	p-value
mu	0.0006	0.000331	1.82	0.069
ar1	-0.296	0.245	-1.21	0.227
ma1	0.345	0.24	1.44	0.151
omega	0.00002	0.000007	3.6	0.000283
alpha1	0.0741	0.0183	4.051	0.000051
beta1	0.8601	0.034	25.53	0

The omega, alpha, and beta were statistically significant because the absolute t statistic values were more than 2. Also, their p-values were less than the test of significance of 0.05. The alpha value was computed as 0.0741 and the beta value as 0.8601. The alpha value is around 0.05; thus, the market was stable. The beta was between 0.85 and 0.98; thus, the persistence was great. The alpha value was low, and the beta value was high; thus, the GARCH volatilities had low vol-of-vol. The stability statistic is 1.9968 indicating a stable market existed.

Heteroskedasticity rarely affects the parameters of Ordinary Least Square (OLS) estimates. However, it has a biasness impact on the variance matrix. To reduce that, the robust standard errors estimates were also computed as shown by table 2 below;

Table 2: Robust standard error estimates

	Estimate	std. Error	t value	p-value
mu	0.0006	0.000396	1.52	0.1278
ar1	-0.296	0.1734	-1.71	0.088
ma1	0.345	0.169	2.036	0.0417
omega	0.00002	0.000034	2.04	0.482
alpha1	0.0741	0.09	0.82	0.41
beta1	0.8601	0.18	4.91	0

The estimates were not much different from those of the optimal parameters. Moving average (MA) effects were statistically significant with robust standard error estimates. Omega, alpha, and beta were also statistically significant.

The information criteria were also computed. The AIC was -5.21, and BIC was -5.19. The less the information criteria, the better the model. The estimates are shown in table 3 below;

Table 3: The information criteria of the AGARCH model

Information criteria	
Akaike	-5.21
Bayes	-5.19
Shibata	-5.21
Hannan- Quinn	-5.2

The autocorrelation between the residuals was tested using the Ljung-Box. The hypothesis was that;

H0: No serial correlation

H1: Serial correlation exists

The researcher obtained the p-value to be 0.873. That value is greater than the alpha at 0.05 significance test; thus, we retain the null hypothesis and conclude no serial correlation in the

residuals. The descriptive statistics of the residuals of the model were calculated. It had a range of 0.004, a mean of 3.701E-04, and a median of 7.39E-05. Table 4 summarizes that.

Table 4: Descriptive statistics of squared-residuals

Min	0
Q1	1.51E-05
Median	7.39E-05
Mean	3.701-4
Q3	2.63E-04
Max	4.00E-02

Figure 3 below shows the squared residuals plotted with the conditional variance. The variance was 2.054213e-06, and the mean was 3.701e-04. The variance is constant all through the series of residuals.

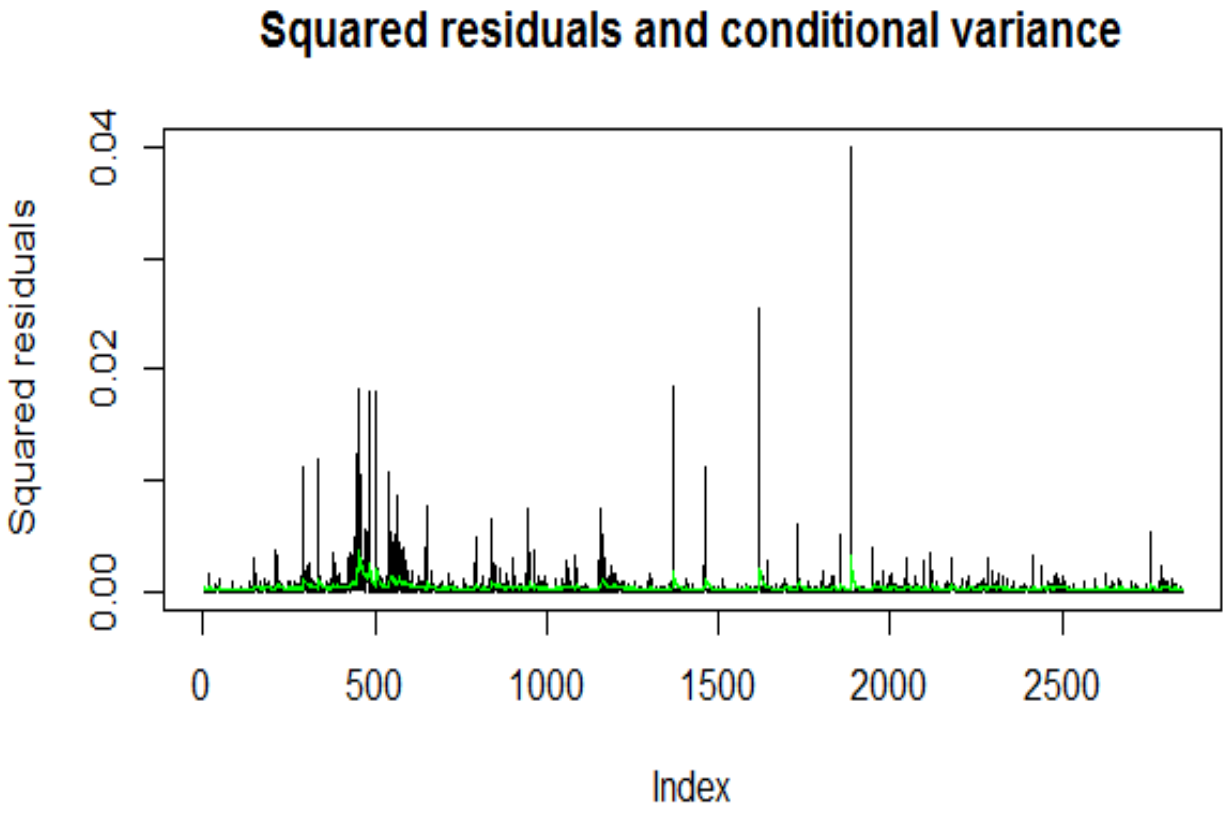

Figure 3: Plot of squared residuals and conditional variance.

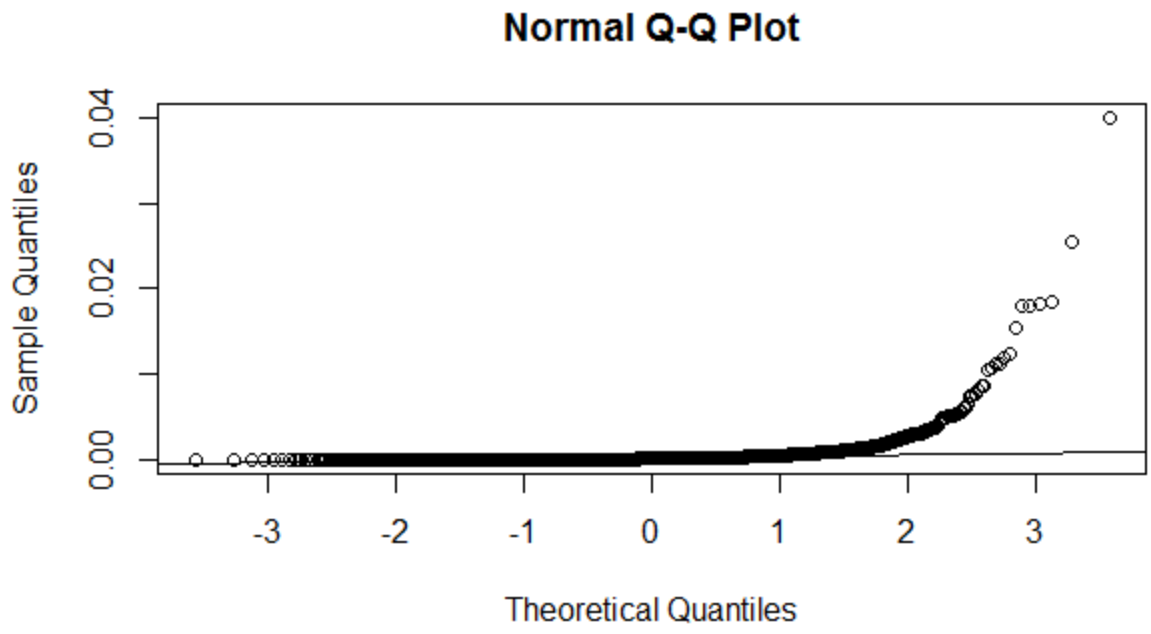

Figure 4: The Q-Q plot of the squared residuals

Forecast

The predictions were made for a future period of 10. From the sixth value, the trend of the series flattens. The values obtained for the series and sigma are shown respectively by the table 5 below;

Table 5: The projected values of the series and the sigma

Time	Series	Sigma
		0.0153
T1	0.00054	8
	0.00062	
T2	3	0.0159
	0.00059	
T3	7	0.0159
	0.00060	0.0161
T4	1	3

	0.00060	0.0162
T5	2	3
	0.00060	
T6	3	0.017
	0.00060	0.0167
T7	3	3
	0.00060	0.0168
T8	3	9
	0.00060	0.0170
T9	3	5
	0.00060	
T10	3	0.0179

The variance of the residuals in the forecast was obtained and plotted as shown by figure 5;

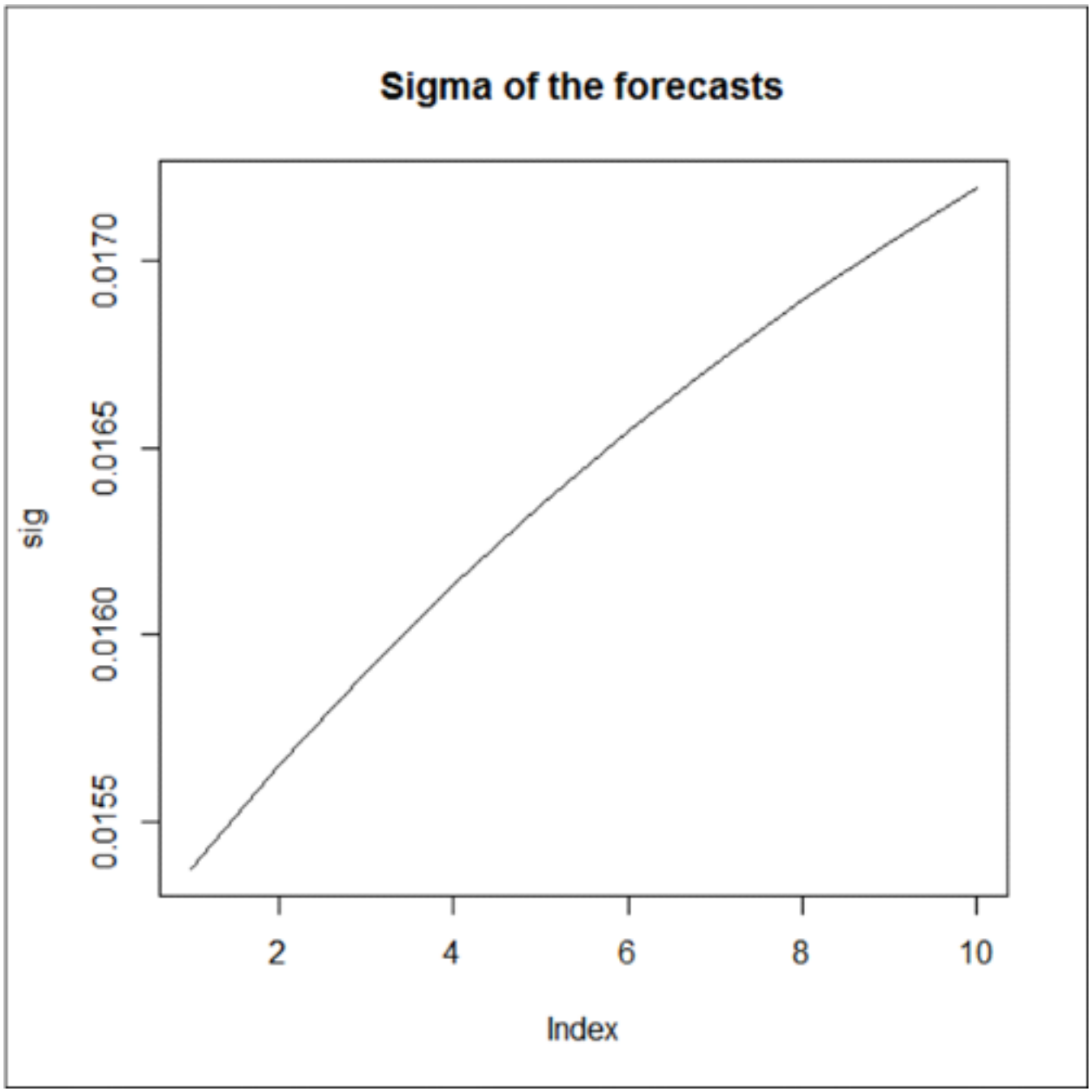

Figure 5: The sigma of the fitted projections

Conclusion

- ♣ The prices of the shares highly fluctuated, but the prices' returns were constant over time.

- The models fitted were GARCH (1, 1) and ARFIMA (1, 0, 1), meaning the autoregressive factor was 1 and the moving average also 1. The model was statistically significant, fitting the returns of the prices of shares.
- The residuals of returns were normally distributed with a mean and variance of almost zero.
- There exists no autocorrelation between the residual values of the returns.
- The projection for the next 10 days indicates that the company will make a profit over that period and has no probability of a loss. The residuals of the projected model were also normally distributed with set mean and sigma values.

References

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