Early Spring 2025
Vol 17, no 1

The Problem Corner

Ivan Retamoso, PhD, *The Problem Corner* Editor
Borough of Manhattan Community College
iretamoso@bmcc.cuny.edu

The Purpose of ***The Problem Corner*** is to give Students and Instructors working independently or together a chance to step out of their "comfort zone" and solve challenging problems. Rather than in the solutions alone, we are interested in methods, strategies, and original ideas following the path toward figuring out the final solutions. We also encourage our Readers to propose new problems. To submit a solution, type it in Microsoft Word, using math type or equation editor, however PDF files are also acceptable. Email your solution as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country. Solutions to posted problem must contain detailed explanation of how the problem was solved. The best solution will be published in a future issue of MTRJ, and correct solutions will be given recognition. To propose a problem, type it in Microsoft Word, using math type or equation editor, email your proposed problem and its solution as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country.

Greetings, problem solvers!

We're delighted to announce that Problems 34 and 35 in *The Problem Corner* have received outstanding solutions, demonstrating both accuracy and ingenuity. By showcasing these exceptional approaches, we hope to inspire others and deepen mathematical understanding across the globe.

Solutions to **problems** from the previous issue.

"Trains" problem.

Problem 34
Proposed by Ivan Retamoso, BMCC, USA.

At noon, Train A is 50 miles west of Train B. Train A moves west at 24 mph, and Train B moves north at 25 mph. At 3 PM, how fast is the distance between the two trains changing?

This content is covered by a Creative Commons license, Attribution-NonCommercial-ShareAlike 4.0 International (CC BY-NC-SA 4.0). This license allows re-users to distribute, remix, adapt, and build upon the material in any medium or format for noncommercial purposes only, and only so long as attribution is given to the creator. If you remix, adapt, or build upon the material, you must license the modified material under identical terms.

First solution to problem 34
By Jahidul Islam, Borough of Manhattan Community College, Bangladesh.

In this concise solution, our solver parametrizes both the distance traveled by the trains and the distance between them in terms of time. By applying differentiation from Calculus, the solution naturally follows.

Since the trains A and B are moving at constant speeds, if S represents the distance traveled and D the distance between the trains after t hours then

$S_A = 50 + 24t$, $S_B = 25t$, and from the theorem of Pythagoras $D = \sqrt{(50 + 24t)^2 + (25t)^2}$

Taking the derivative of D with respect to t

$$D'(t) = \frac{1201t + 1200}{\sqrt{1201t^2 + 2400t + 2500}}$$

Since there are 3 hours from noon to 3 PM then for $t = 3$

$$D'(3) = 33.54\, mph$$

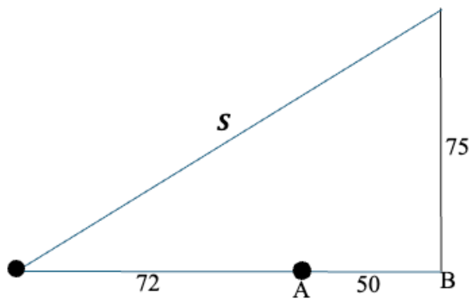
Therefore, the distance between the two trains is changing at approximately 33.54 mph at 3 PM.

Second solution to problem 34
By Dr. Abdullah Kurudirek, Mathematics Ed. Department, Tishk International University.

In this alternative solution, our solver first establishes a relationship between the distances traveled by the trains using the Pythagorean theorem. By differentiating this equation with respect to time, our solver derives a relationship between the rates of change which leads to the final answer. A diagram aids in visualizing the solution.

This content is covered by a Creative Commons license, Attribution-NonCommercial-ShareAlike 4.0 International (CC BY-NC-SA 4.0). This license allows re-users to distribute, remix, adapt, and build upon the material in any medium or format for noncommercial purposes only, and only so long as attribution is given to the creator. If you remix, adapt, or build upon the material, you must license the modified material under identical terms.

This problem may be solved using related rates in calculus. The distance of Train A is $50\ ml$ and the distance of Train B is $0\ ml$ from its starting position (at noon). Let $\boldsymbol{S}$ be the distance between the two trains.



After 3 hours $x = 50 + 24.(3) = 122\ ml$ and $y = 0 + 25.(3) = 75\ ml$
Using the Pythagorean Theorem $S^2 = x^2 + y^2$ and substituting values $S^2 = 122^2 + 75^2$ then

$$S = \sqrt{20509}\ ml$$

Now, differentiating implicitly with respect to t and simplifying $S^2 = x^2 + y^2$ we get

$$S\frac{ds}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$

After substituting known values

$$\sqrt{20509}\frac{ds}{dt} = 122.24 + 75.25$$

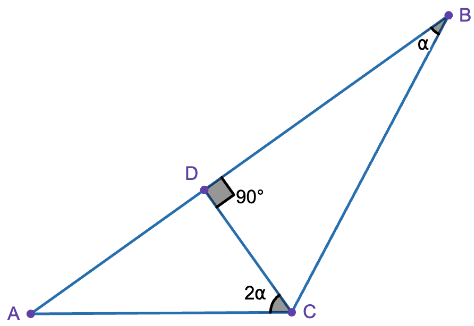
$$\frac{ds}{dt} \approx 33.5\ ml/h$$

The distance between the two trains is increasing at approximately $33.5\ ml/h$ at 3 PM.

"Tricky Area of a Triangle" problem.

Problem 35
Proposed by Ivan Retamoso, BMCC, USA.
In the given figure, the length of BD is 12 $inches$, and AC measures 10 $inches$. Determine the area of triangle ABC.

This content is covered by a Creative Commons license, Attribution-NonCommercial-ShareAlike 4.0 International (CC BY-NC-SA 4.0). This license allows re-users to distribute, remix, adapt, and build upon the material in any medium or format for noncommercial purposes only, and only so long as attribution is given to the creator. If you remix, adapt, or build upon the material, you must license the modified material under identical terms.



Solution to problem 35

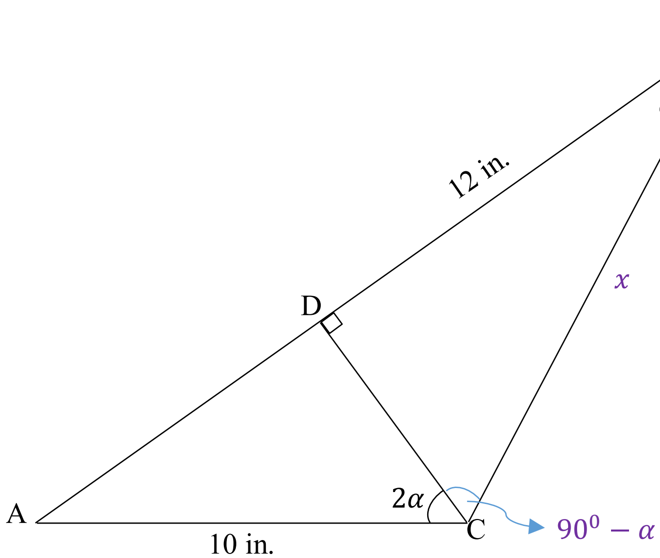
By Dr. Abdullah Kurudirek, Mathematics Ed. Department, Tishk International University.

Our solver presents two solutions: the first uses trigonometry, applying sine and cosine formulas to compute the area of the triangle, while the second relies on an ingenious auxiliary geometric construction. Both solutions are accompanied by helpful graphs.

Method 1. Let the side BC be x inches. Since the sum of the interior angles of triangle BDC is 180^0, angle C will be $90^0 - \alpha$. The area of a triangle using trigonometry, we can write the equation below. By using reduction formula $\left[Sin\left(\frac{\pi}{2} + \alpha\right) = Cos\alpha \right]$we find the area of triangle ABC to be $60\ in.^2$

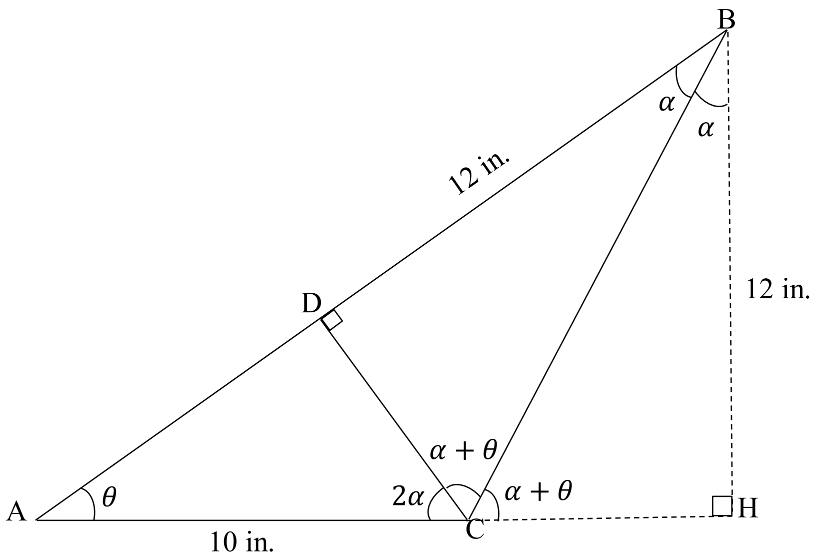
Area of triangle **ABC** $= \frac{1}{2} AC.BC.SinC = \frac{1}{2} 10.x.Sin(90^0 + \alpha)$

Area of triangle **ABC** $= \frac{1}{2} 10.x.Cos(\alpha) = \frac{1}{2} 10.x.\frac{12}{x} = 60\ in.^2$



This content is covered by a Creative Commons license, Attribution-NonCommercial-ShareAlike 4.0 International (**CC BY-NC-SA 4.0**). This license allows re-users to distribute, remix, adapt, and build upon the material in any medium or format for noncommercial purposes only, and only so long as attribution is given to the creator. If you remix, adapt, or build upon the material, you must license the modified material under identical terms.

Method 2. Let us draw a perpendicular from point B to the base AC. Then, it will be seen that the angles at B and C are as shown in the figure. After this, we can easily calculate the area using our classic formula, which will be as follows.



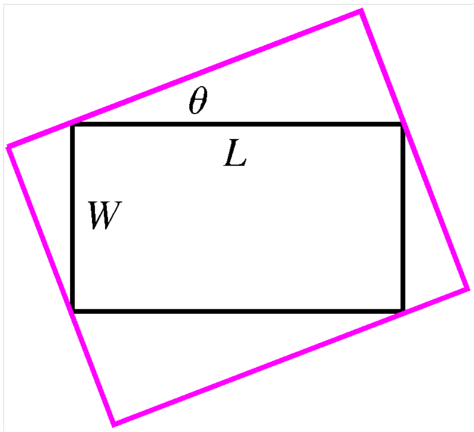
$$\text{Area of triangle } \mathbf{ABC} = \frac{1}{2}AC.BH = \frac{1}{2}10.12 = 60\ in.^2$$

I'm happy to hear that you enjoyed solving problems 34 and 35 and have refined your mathematical approach. Let's keep up the great work and tackle the next set of challenges to further enhance your skills!

Problem 36
Proposed by Ivan Retamoso, BMCC, USA.

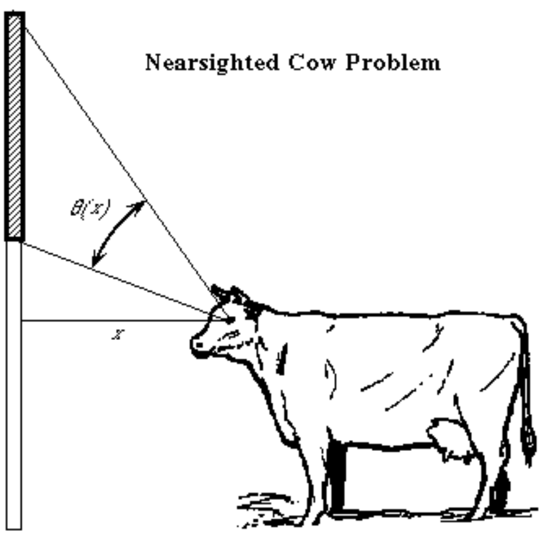
Find the maximum area of a rectangle that can be circumscribed about a given rectangle with length L and width W. (Your answer may depend on L and W.)

This content is covered by a Creative Commons license, Attribution-NonCommercial-ShareAlike 4.0 International (CC BY-NC-SA 4.0). This license allows re-users to distribute, remix, adapt, and build upon the material in any medium or format for noncommercial purposes only, and only so long as attribution is given to the creator. If you remix, adapt, or build upon the material, you must license the modified material under identical terms.



Problem 37
Proposed by Ivan Retamoso, BMCC, USA.

A rectangular billboard 5 feet in height stands in a field so that its bottom is 10 feet above the ground. A nearsighted cow with eye level at 4 feet above the ground stands x feet from the billboard. Express θ, the vertical angle subtended by the billboard at her eye, in terms of x. Then find the distance x_0 the cow must stand from the billboard to maximize θ.



This content is covered by a Creative Commons license, Attribution-NonCommercial-ShareAlike 4.0 International (CC BY-NC-SA 4.0). This license allows re-users to distribute, remix, adapt, and build upon the material in any medium or format for noncommercial purposes only, and only so long as attribution is given to the creator. If you remix, adapt, or build upon the material, you must license the modified material under identical terms.