Intermediate Value Theorem

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) Use the Intermediate Value Theorem to prove that $-2x^4 + 6x^3 + 5x - 2 = 0$ has a solution between 3 and 4. 1) _____

2) Use the Intermediate Value Theorem to prove that $7x^3 + 9x^2 - 6x - 5 = 0$ has a solution between -2 and -1. 2) _____

3) Use the Intermediate Value Theorem to prove that $x(x-2)^2 = 2$ has a solution between 1 and 3. 3) _____

4) Use the Intermediate Value Theorem to prove that $4 \sin x = x$ has a solution between $\frac{\pi}{2}$ and π. 4) _____

1) Let $f(x) = -2x^4 + 6x^3 + 5x - 2$ and let $y_0 = 0$. $f(3) = 13$ and $f(4) = -110$. Since f is continuous on $[3, 4]$ and since $y_0 = 0$ is between $f(3)$ and $f(4)$, by the Intermediate Value Theorem, there exists a c in the interval $(3, 4)$ with the property that $f(c) = 0$. Such a c is a solution to the equation $-2x^4 + 6x^3 + 5x - 2 = 0$.
2) Let $f(x) = 7x^3 + 9x^2 - 6x - 5$ and let $y_0 = 0$. $f(-2) = -13$ and $f(-1) = 3$. Since f is continuous on $[-2, -1]$ and since $y_0 = 0$ is between $f(-2)$ and $f(-1)$, by the Intermediate Value Theorem, there exists a c in the interval $(-2, -1)$ with the property that $f(c) = 0$. Such a c is a solution to the equation $7x^3 + 9x^2 - 6x - 5 = 0$.
3) Let $f(x) = x(x-2)^2$ and let $y_0 = 2$. $f(1) = 1$ and $f(3) = 3$. Since f is continuous on $[1, 3]$ and since $y_0 = 2$ is between $f(1)$ and $f(3)$, by the Intermediate Value Theorem, there exists a c in the interval $(1, 3)$ with the property that $f(c) = 2$. Such a c is a solution to the equation $x(x-2)^2 = 2$.
4) Let $f(x) = \frac{\sin x}{x}$ and let $y_0 = \frac{1}{4}$. $f\left(\frac{\pi}{2}\right) \approx 0.6366$ and $f(\pi) = 0$. Since f is continuous on $\left[\frac{\pi}{2}, \pi\right]$ and since $y_0 = \frac{1}{4}$ is between $f\left(\frac{\pi}{2}\right)$ and $f(\pi)$, by the Intermediate Value Theorem, there exists a c in the interval $\left(\frac{\pi}{2}, \pi\right)$, with the property that $f(c) = \frac{1}{4}$. Such a c is a solution to the equation $4 \sin x = x$.